Introduction to calculating the digitally controlled PID filter

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http://www.optoelec-engineering.com/index_en.html

1. Introduction

We supply the application sample code of DSP / TMS320F28335 as to the digitally controlled PID filter via our "***DSP F28335 Basic Control Platform C-Programming Code Sets***".

We provide another detail documents about feedback control. Please check the following document.

"***Introduction to feed back control.pdf***"

The sample application of the digitally controlled PID filter is the incomplete integral type such as Fig- 1. We get the transfer function of Fig- 1 with replacing these variables such as eq.- 1 for the Fig-2 and eq.-17 of "***Introduction to feed back control.pdf***".

R3→R1, R2→open, C2→open, R1→R2, C1→C1, Vref→0

eq.- 1

Additionally, we consider the open-loop gain Av of OP-Amp. Thus, we make the equation as to transfer function for Fig- 1.

$$i_1 = i_3 = \frac{V_{in} - V_c}{R_1} = \frac{V_c - V_{out}}{R_2 + \dfrac{1}{j\omega C_1}}$$

eq.- 2

$$V_{out} = A_V \times (0 - V_c)$$

eq.- 3

Substitute eq.- 2 to eq.- 3. Here, s=jω=j×2πf.

$$\frac{V_{in} + \dfrac{V_{out}}{A_V}}{R_1} = \frac{-\dfrac{V_{out}}{A_V} - V_{out}}{R_2 + \dfrac{1}{sC_1}}$$

$$-\left(R_2 + \frac{1}{sC_1}\right)\cdot\left(\frac{V_{in}}{R_1} + \frac{V_{out}}{A_V R_1}\right) = \left(\frac{1}{A_V} + 1\right)\cdot V_{out}$$

$$-\frac{R_2}{R_1}\cdot\left(1 + \frac{1}{sC_1R_2}\right)\cdot\left(V_{in} + \frac{V_{out}}{A_V}\right) = \left(\frac{1}{A_V} + 1\right)\cdot V_{out}$$

$$-\frac{R_2}{R_1}\cdot\left(1 + \frac{1}{sC_1R_2}\right)\cdot V_{in} = \left(\frac{1}{A_V} + 1\right)\cdot V_{out} + \frac{R_2}{R_1}\cdot\left(1 + \frac{1}{sC_1R_2}\right)\cdot\frac{V_{out}}{A_V}$$

Therefore, we get the transfer function of PID filter Fig- 1.

$$
\begin{aligned}
G_{PID}(s) = \frac{V_{out}}{V_{in}} &= \frac{-\dfrac{R_2}{R_1}\cdot\left(1+\dfrac{1}{sC_1R_2}\right)}{\dfrac{R_2}{R_1}\cdot\left(1+\dfrac{1}{sC_1R_2}\right)\cdot\dfrac{1}{A_V}+\dfrac{1}{A_V}+1} \\
&= \frac{-\dfrac{R_2}{R_1}\cdot sC_1R_2-\dfrac{R_2}{R_1}}{\dfrac{R_2}{R_1}\cdot sC_1R_2\cdot\dfrac{1}{A_V}+\dfrac{R_2}{R_1}\cdot\dfrac{1}{A_V}+\left(\dfrac{1}{A_V}+1\right)\cdot sC_1R_2} \\
&= \frac{-\dfrac{C_1R_2}{R_1}\cdot s-\dfrac{1}{R_1}}{\dfrac{C_1R_2}{R_1}\cdot\dfrac{1}{A_V}\cdot s+\dfrac{1}{R_1}\cdot\dfrac{1}{A_V}+\left(\dfrac{1}{A_V}+1\right)\cdot sC_1} \\
&= \frac{-C_1R_2\cdot s-1}{C_1R_2\cdot\dfrac{1}{A_V}\cdot s+\dfrac{1}{A_V}+\left(\dfrac{1}{A_V}+1\right)\cdot sC_1R_1} \\
&= -\frac{sC_1R_2+1}{s\cdot\left((C_1R_2+C_1R_1)\dfrac{1}{A_V}+C_1R_1\right)+\dfrac{1}{A_V}} \\
&= -\frac{sC_1R_2+1}{s\cdot C_1\left((R_2+R_1)\dfrac{1}{A_V}+R_1\right)+\dfrac{1}{A_V}} \\
&= \frac{-A_V\cdot(s\cdot C_1R_2+1)}{s\cdot C_1\left(R_2+(1+A_V)\cdot R_1\right)+1}
\end{aligned}
$$

eq.- 4

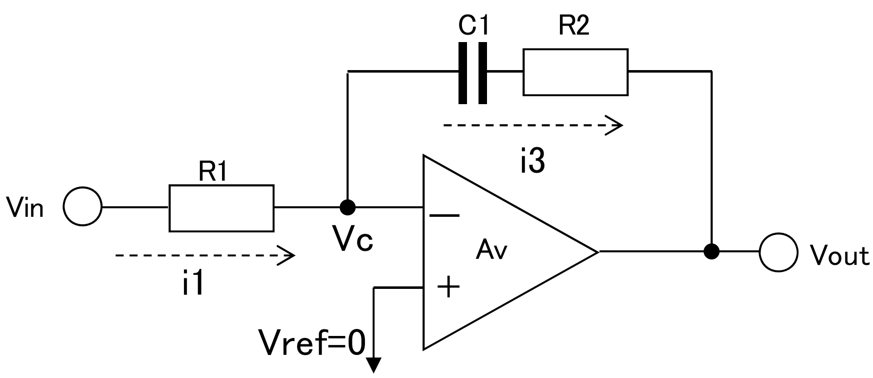

Fig- 1 PID filter of the incomplete integral type provided in
"*DSP F28335 Basic Control Platform C-Programming Code Sets*"

Here, we get a zero-point and a pole-point from eq.- 4.

Zero-point $$f_{c2} = \frac{1}{2\pi C_1 R_2}$$

eq.- 5

Pole-point $$f_{c1} = \frac{1}{2\pi C_1 \left(R_2 + (1 + A_v)R_1\right)} \approx \frac{1}{2\pi A_v C_1 R_1}$$

eq.- 6

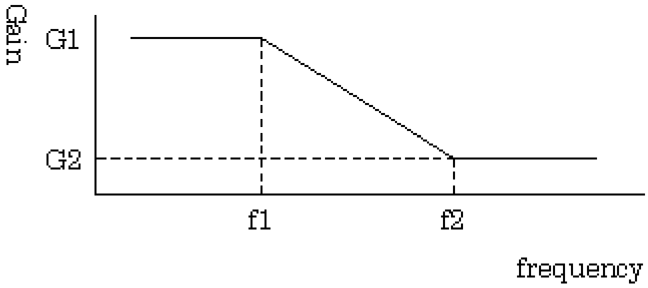

Fig- 2 Transfer characteristics of PID filter Fig- 1

We calculate the amplitude term and the phase term of eq.- 4.

First, we replace eq.- 4 by eq.- 7.

$$P = -A_v C_1 R_2 \qquad Q = C_1\left(R_2 + (1 + A_v)R_1\right)$$

eq.- 7

Then, we get eq.- 8.

$$G_{PID}(s) = \frac{V_{out}}{V_{in}} = \frac{-A_V \cdot (s \cdot C_1 R_2 + 1)}{s \cdot C_1\left(R_2 + (1 + A_V) \cdot R_1\right) + 1} = \frac{s \cdot P - A_v}{s \cdot Q + 1}$$

eq.- 8

And more, we expand eq.- 8.

$$G_{PID}(s) = \frac{V_{out}}{V_{in}} = \frac{s \cdot P - A_v}{s \cdot Q + 1} = \frac{(s \cdot P - A_v) \cdot (s \cdot Q - 1)}{(s \cdot Q + 1) \cdot (s \cdot Q - 1)} = \frac{s^2 \cdot PQ - s \cdot (A_v Q + P) + A_v}{s^2 \cdot Q^2 - 1}$$
$$= \frac{-\omega^2 \cdot PQ - s \cdot (A_v Q + P) + A_v}{-\omega^2 \cdot Q^2 - 1} = \frac{\omega^2 \cdot PQ - A_v}{\omega^2 \cdot Q^2 + 1} + j \times \omega \cdot \frac{A_v Q + P}{\omega^2 \cdot Q^2 + 1}$$

eq.- 9

The amplitude component (Gain) is

$$Gain\left[V/V\right] = \sqrt{\left(\frac{\omega^2 \cdot PQ - A_v}{\omega^2 \cdot Q^2 + 1}\right)^2 + \left(\omega \cdot \frac{A_v Q + P}{\omega^2 \cdot Q^2 + 1}\right)^2} = \frac{\sqrt{\left(\omega^2 \cdot PQ - A_v\right)^2 + \omega^2 \cdot (A_v Q + P)^2}}{\omega^2 \cdot Q^2 + 1}$$

eq.- 10

The phase component is

$$Phsase[rad] = \tan^{-1}\left(\frac{\omega \cdot \dfrac{A_v Q + P}{\omega^2 \cdot Q^2 + 1}}{\dfrac{\omega^2 \cdot PQ - A_v}{\omega^2 \cdot Q^2 + 1}} \right) = \tan^{-1}\left(\frac{\omega \cdot (A_v Q + P)}{(\omega^2 \cdot PQ - A_v)} \right)$$

eq.- 11

2. Principle of Digitally controlled PID filter

We start to discretize the transfer function eq.- 9 of analog PID filter by Bilinear Transform eq.- 12.

Bilinear Transform ($s \rightarrow z^{-1}$) $\quad s = \frac{2}{T_s} \cdot \frac{1 - z^{-1}}{1 + z^{-1}}$

eq.- 12

Here each parameter is,

Analog complex angular frequency s=j×ω= j×2πf

Ts ; a time of sampling period

z^{-1} ; delay-operator eq.- 13

$$z^{-1} = e^{-j \cdot \Omega} = \cos(\Omega) - j \cdot \sin(\Omega)$$

eq.- 13

Ω is digital angular frequency.

$$\Omega = 2\tan^{-1}\left(\frac{T_s \cdot \omega}{2} \right)$$

eq.- 14

Thus, analog complex angular frequency s and analog angular frequency ω is

$$s = j\omega = j \cdot \frac{2}{T_s} \times \tan\left(\frac{\Omega}{2} \right) \qquad \omega = \frac{2}{T_s} \cdot \tan\left(\frac{\Omega}{2} \right)$$

eq.- 15

Substitute eq.- 12 into eq.- 8.

$$G_{PID}(s) = \frac{s \cdot P - A_v}{s \cdot Q + 1} = \frac{\dfrac{2}{T_s} \cdot \dfrac{1 - z^{-1}}{1 + z^{-1}} \cdot P - A_v}{\dfrac{2}{T_s} \cdot \dfrac{1 - z^{-1}}{1 + z^{-1}} \cdot Q + 1} = \frac{\dfrac{2}{T_s} P \cdot (1 - z^{-1}) - A_v \cdot (1 + z^{-1})}{\dfrac{2}{T_s} Q \cdot (1 - z^{-1}) + (1 + z^{-1})}$$

eq.- 16

Here, replace new parameter of to eq.- 16

$$\xi = \frac{2P}{T_s} \qquad \chi = \frac{2Q}{T_s}$$

eq.- 17

Thus

$$G_{PID}(z)=\frac{OUT(z)}{IN(z)}=\frac{\frac{2}{T_s}P\cdot\left(1-z^{-1}\right)-A_v\cdot\left(1+z^{-1}\right)}{\frac{2}{T_s}Q\cdot\left(1-z^{-1}\right)+\left(1+z^{-1}\right)}=\frac{\xi\cdot\left(1-z^{-1}\right)-A_v\cdot\left(1+z^{-1}\right)}{\chi\cdot\left(1-z^{-1}\right)+\left(1+z^{-1}\right)}$$

eq.- 18

Expand the eq.- 18.

$$OUT(z)\cdot\left\{\chi\cdot\left(1-z^{-1}\right)+\left(1+z^{-1}\right)\right\}=IN(z)\cdot\left\{\xi\cdot\left(1-z^{-1}\right)-A_v\cdot\left(1+z^{-1}\right)\right\}$$

$$(\chi+1)\cdot OUT(z)+(-\chi+1)\cdot z^{-1}OUT(z)=(\xi-A_v)\cdot IN(z)-(\xi+A_v)\cdot z^{-1}IN(z)$$

eq.- 19

Therefore,

$$OUT(z)=\frac{\xi-A_v}{\chi+1}\cdot IN(z)-\frac{\xi+A_v}{\chi+1}\cdot z^{-1}IN(z)+\frac{\chi-1}{\chi+1}\cdot z^{-1}OUT(z)$$
$$=a\times IN(z)+b\times z^{-1}IN(z)+c\times z^{-1}OUT(z)$$

eq.- 20

Here, a, b, c constants are

$$a=\frac{\xi-A_v}{\chi+1}\qquad b=-\frac{\xi+A_v}{\chi+1}\qquad c=\frac{\chi-1}{\chi+1}$$

eq.- 21

z^{-1} is delay-operator. The z^{-1} • IN(z) is previous sampling input data with one-time sampling delay, the z^{-1} • OUT(z) is previous sampling output data with one-time sampling delay.

The circuit diagram of eq.- 20 are wrote on Fig- 3.

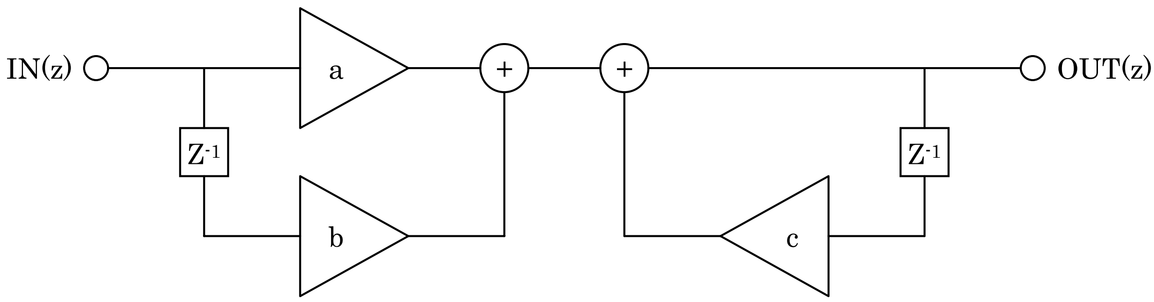

Fig- 3 Diagram of digitally controlled PID filter

We transform the equation eq.- 18 to a format of digital angular frequency Ω. Substitute eq.- 13 to eq.- 18. We get

$$G(z)=\frac{\{\xi(1-\cos(\Omega))-A_v(1+\cos(\Omega))\}+j\times\sin(\Omega)\times(\xi+A_v)}{\{\chi(1-\cos(\Omega))+(1+\cos(\Omega))\}+j\times\sin(\Omega)\times(\chi-1)}=\frac{D+j\cdot E}{F+j\cdot G}=\frac{DF+EG}{F^2+G^2}+j\times\frac{EF-DG}{F^2+G^2}$$

eq.- 22

Here,

$$D = \xi\left(1-\cos(\Omega)\right) - A_v\left(1+\cos(\Omega)\right)$$
$$E = \sin(\Omega)\times\left(\xi + A_v\right)$$
$$F = \chi\left(1-\cos(\Omega)\right) + \left(1+\cos(\Omega)\right)$$
$$G = \sin(\Omega)\times\left(\chi - 1\right)$$

eq.- 23

3. How to use our "***Calculator of digitally controlled PID filter***". (Excel files)

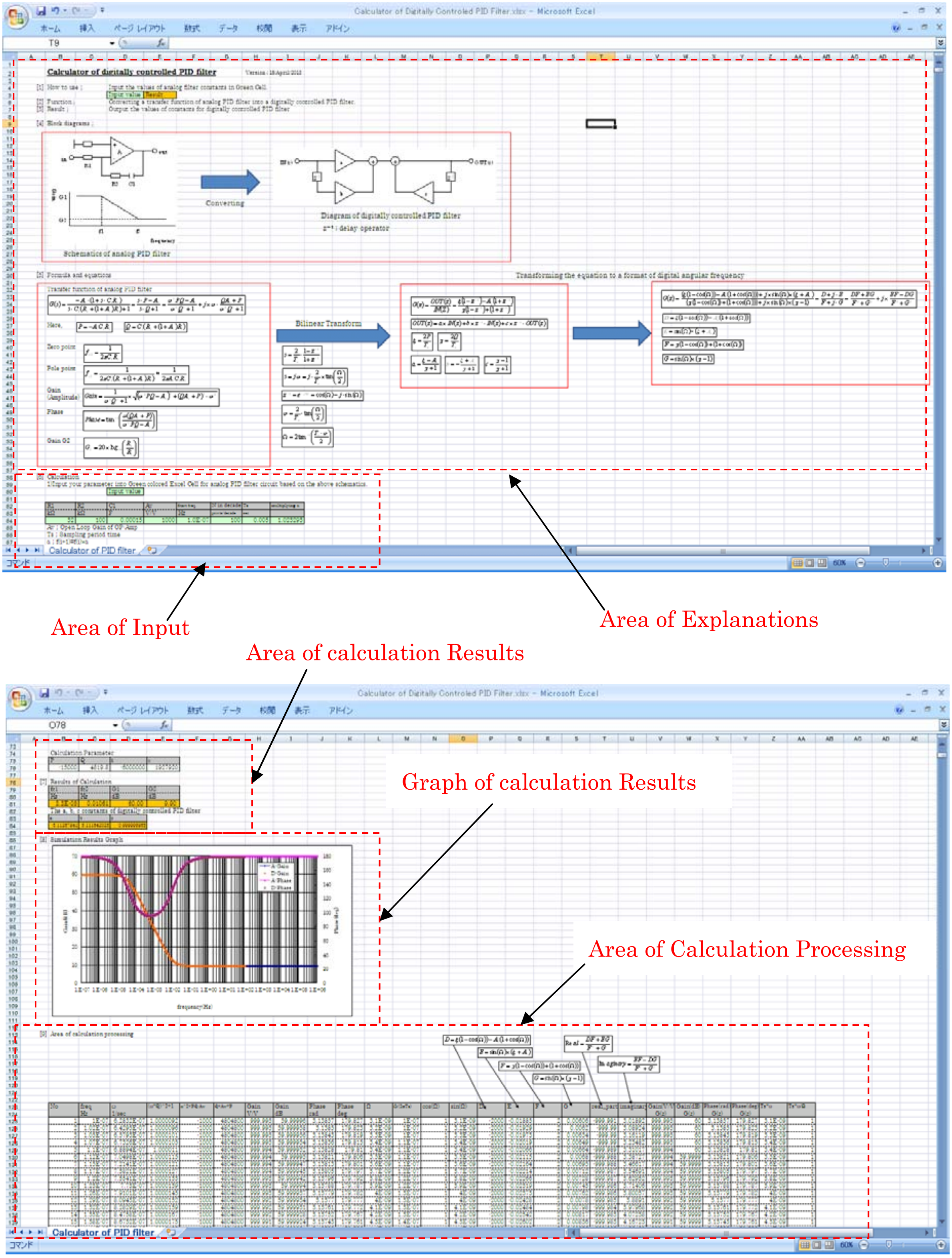

Fig- 4 Whole view of Excel Calculator

[1] Input your values of analog filter constants in Green Cell of excel calculator.

[2] User can get the results of calculator.

[3] Please read the comments of the excel calculator.

4. Our websales site

We sell the "*Calculator of digitally controlled PID filter* " and its document "*Introduction to calculating the digitally controlled PID filter*". Please visit our websales site.

http://www.optoelec-engineering.com/websale/websale_en.html