Introduction to feed back control

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1. Introduction

It is shown the diagram of the feed back control in Fig- 1. The feed back control consists of error amplifier, compensator, controlled objection, feed back block.

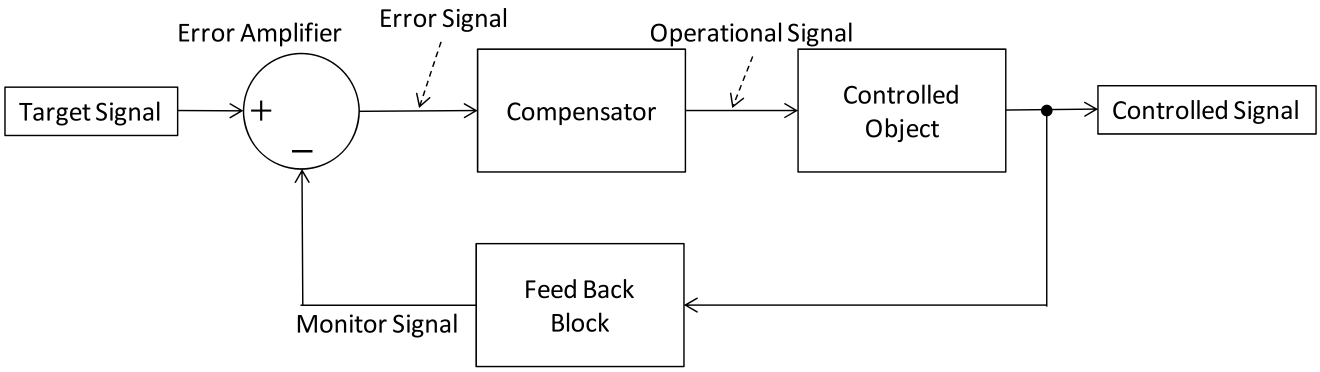

Fig- 1 diagram of feed back control

The compensator applies the PID filter which has a first-order lag element. P means what the operational signal is generated by the amplified quantity proportional to the error signal. I means what the operational signal is generated by the amplified quantity proportional to the integration of the error signal. Similarly D means derivative.

The time-dependency equations as to the operational signal y(t) is

$$y(t) = K_p\left(\varepsilon(t) + \frac{1}{T_i}\int \varepsilon(t)dt + T_d\frac{d}{dt}\varepsilon(t)\right) \quad , \ \varepsilon(t)\text{: error signal}$$

eq.- 1

We apply Laplace transform to eq.- 1. Thus,

$$\begin{aligned} L[y(t)] &= K_p L[\varepsilon(t)] + \frac{K_p}{T_i}\cdot L\left[\int \varepsilon(t)dt\right] + K_p T_d \cdot L\left[\frac{d}{dt}\varepsilon(t)\right] \\ &= K_p L[\varepsilon(t)] + \frac{K_p}{T_i}\cdot\frac{L[\varepsilon(t)]}{s} + K_p T_d\cdot s\cdot L[\varepsilon(t)] \\ &= K_p\left(1 + \frac{1}{T_i\cdot s} + T_d\cdot s\right)\cdot L[\varepsilon(t)] \end{aligned}$$

eq.- 2

Here, s=jω=j×2πf. Therefore, the transfer function of PID filter is

$$G_{PID}(s) = \frac{L[y(t)]}{L[\varepsilon(t)]} = K_p\left(1 + \frac{1}{T_i\cdot s} + T_d\cdot s\right)$$

eq.- 3

We can transform eq.- 3 into the term of product form such as eq.- 4.

$$G'_{PID}(s) = K'_p \left(1 + \frac{1}{T'_i \cdot s}\right)(1 + T'_d \cdot s)$$

eq.- 4

We can prove the relation between eq.- 3 and eq.- 4. We develop eq.- 4.

$$G'_{PID}(s) = K'_p \left(1 + \frac{1}{T'_i \cdot s}\right)(1 + T'_d \cdot s) = K'_p \left(1 + \frac{1}{T'_i \cdot s} + T'_d \cdot s + \frac{T'_d}{T'_i}\right)$$

$$= K'_p \left(1 + \frac{T'_d}{T'_i}\right)\left(1 + \frac{1}{T'_i\left(1 + \frac{T'_d}{T'_i}\right) \cdot s} + \frac{T'_d}{1 + \frac{T'_d}{T'_i}} \cdot s\right)$$

eq.- 5

We get the relation between eq.- 3 and eq.- 4.

$$K_p = K'_p \left(1 + \frac{T'_d}{T'_i}\right), \quad T_i = T'_i \left(1 + \frac{T'_d}{T'_i}\right), \quad T_d = \frac{T'_d}{1 + \frac{T'_d}{T'_i}}$$

eq.- 6

Then we refer to the dumping factor. The dumping factor η absorbs the derivative effect in order to avoid unstability of feed back loop. The derivative term generates the divergence of PID transfer function with limitation $s \to \infty$. So we apply the dumping factor η to eq.- 3.

$$G_{PID}(s) = K_p \left(1 + \frac{1}{T_i \cdot s} + \frac{T_d}{1 + \eta T_d \cdot s} \cdot s\right)$$

eq.- 7

or,

$$G_{PID}(s) = K_p \left(1 + \frac{1}{T_i \cdot s} + T_d \cdot s\right) \cdot \frac{1}{1 + \eta T_d \cdot s}$$

$$= K'_p \left(1 + \frac{1}{T'_i \cdot s}\right)(1 + T'_d \cdot s) \cdot \frac{1}{1 + \eta T_d \cdot s}$$

eq.- 8

Here, η is nearly equal to 0.05 to 0.2.

2. Equivalent analog circuit for PID filter (compensator)

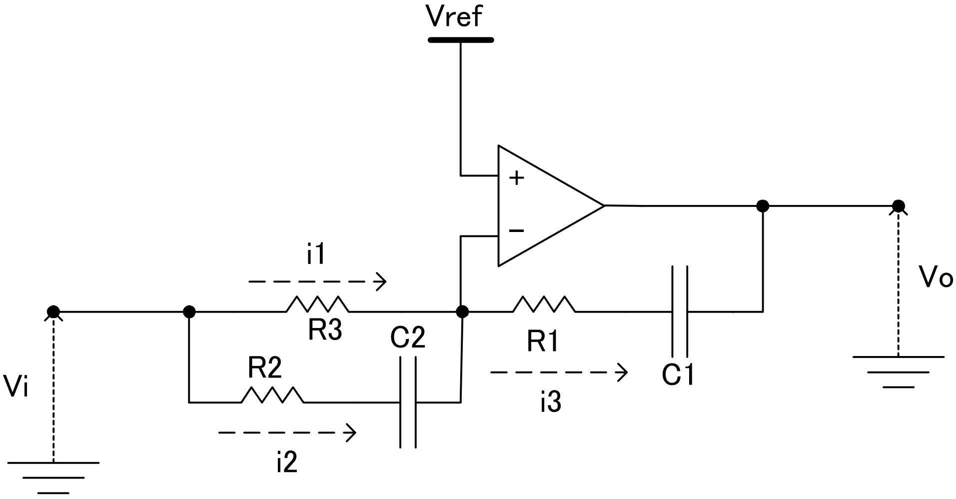

Fig- 2 Equivalent analog circuit for PID filter

It is shown the equivalent OP-Amp model for PID filter at Fig- 2.

2.1 Derivation of transfer function as to OP-Amp model for PID filter

; Solver via time-dependency equation (Assuming Vref = 0)

We make the equation from input signal node to invert input pin of OP-Amp. The invert input pin of OP-Amp is imaginary earth because of Vref = 0.

$$V_i(t) = R_2 i_2 + \frac{1}{C_2}\int i_2 dt = R_3 i_1$$

eq.- 9

We make the equation from invert input pin of OP-Amp to output signal node.

$$0V - V_o = i_3 R_1 + \frac{1}{C_1}\int i_3 dt$$

eq.- 10

We apply Laplace transform to eq.- 9.

$$L[V_i(t)] = L[R_2 i_2] + L\left[\frac{1}{C_2}\int i_2 dt\right] = L[R_3 i_1]$$

$$= R_2 L[i_2] + \frac{1}{C_2}\frac{L[i_2]}{s} = R_3 L[i_1]$$

eq.- 11

Thus,

$$L[i_1] = \frac{L[V_i(t)]}{R_3}$$

eq.- 12

$$L[i_2] = \frac{L[V_i(t)]}{R_2 + \frac{1}{C_2 s}}$$

eq.- 13

We apply Laplace transform to eq.- 10.

$$-L[V_o(t)] = R_1 L[i_3] + \frac{1}{C_1}\frac{L[i_3]}{s} = \left(R_1 + \frac{1}{C_1 s}\right)L[i_3]$$

eq.- 14

Thus,

$$L[i_3] = -\frac{L[V_o(t)]}{\left(R_1 + \frac{1}{C_1 s}\right)}$$

eq.- 15

Here, we remember the next equation.

$$L[i_3] = L[i_1] + L[i_2]$$

eq.- 16

Substitute eq.- 13, eq.- 14, eq.- 15 into eq.- 16.

$$-\frac{L[V_o(t)]}{\left(R_1 + \frac{1}{C_1 s}\right)} = \frac{L[V_i(t)]}{R_3} + \frac{L[V_i(t)]}{R_2 + \frac{1}{C_2 s}}$$

Therefore,

$$\frac{L[V_o(t)]}{L[V_i(t)]} = -\left(\frac{1}{R_3} + \frac{1}{R_2 + \frac{1}{C_2 s}}\right)\cdot\left(R_1 + \frac{1}{C_1 s}\right)$$

$$= -R_1\left(\frac{1}{R_3} + \frac{C_2 s}{R_2 C_2 s + 1}\right)\cdot\left(1 + \frac{1}{R_1 C_1 s}\right)$$

We get the transfer function of OP-Amp model for PID filter at Fig- 2. (Vref = 0)

$$G(s) = -\frac{R_1}{R_3}\left(1 + \frac{R_3 C_2 s}{R_2 C_2 s + 1}\right)\cdot\left(1 + \frac{1}{R_1 C_1 s}\right)$$

eq.- 17

2.2 Derivation of transfer function as to OP-Amp model for PID filter

; Solver via Ohm-law equation (Assuming Vref $\neq$ 0)

We make the equation such as next via Ohm-law.

$$V_i - V_{ref} = i_1 R_3 = i_2 \left(R_2 + \frac{1}{j\omega \cdot C_2} \right)$$

eq.- 18

$$V_{ref} - V_o = i_3 \left(R_1 + \frac{1}{j\omega \cdot C_1} \right)$$

eq.- 19

$$i_3 = i_1 + i_2$$

eq.- 20

Substitute eq.- 18, eq.- 19 into eq.- 20.

$$\frac{V_{ref} - V_o}{R_1 + \frac{1}{j\omega \cdot C_1}} = \frac{V_i - V_{ref}}{R_3} + \frac{V_i - V_{ref}}{R_2 + \frac{1}{j\omega \cdot C_2}}$$

$$\frac{V_{ref} - V_o}{V_i - V_{ref}} = \left(R_1 + \frac{1}{j\omega \cdot C_1} \right) \cdot \left(\frac{1}{R_3} + \frac{1}{R_2 + \frac{1}{j\omega \cdot C_2}} \right)$$

$$= \left(R_1 + \frac{1}{s \cdot C_1} \right) \cdot \left(\frac{1}{R_3} + \frac{1}{R_2 + \frac{1}{s \cdot C_2}} \right)$$

$$= \left(R_1 + \frac{1}{s \cdot C_1} \right) \cdot \left(\frac{1}{R_3} + \frac{s \cdot C_2}{s \cdot C_2 R_2 + 1} \right)$$

$$= \frac{R_1}{R_3} \left(1 + \frac{1}{s \cdot C_1 R_1} \right) \cdot \left(1 + \frac{s \cdot C_2 R_3}{s \cdot C_2 R_2 + 1} \right)$$

eq.- 21

Therefore,

$$G(s) = \frac{V_o - V_{ref}}{V_i - V_{ref}} = -\frac{R_1}{R_3} \left(1 + \frac{1}{s \cdot C_1 R_1} \right) \cdot \left(1 + \frac{s \cdot C_2 R_3}{s \cdot C_2 R_2 + 1} \right)$$

eq.- 22

2.3 Relation between the circuit constant of OP-Amp model and the coefficient of PID compensator

We expand the eq.- 17.

$$G(s) = -\frac{R_1}{R_3}\left(1+\frac{R_3C_2s}{R_2C_2s+1}\right)\cdot\left(1+\frac{1}{R_1C_1s}\right)$$

$$= -\frac{R_1}{R_3}\left(1+\frac{1}{R_1C_1s}\right)\frac{1+C_2(R_3+R_2)s}{R_2C_2s+1}$$

eq.- 23

Compare eq.- 8 to eq.- 23.

$$K'_p = -\frac{R_1}{R_3}$$

$$T'_i = R_1C_1$$

$$T'_d = C_2(R_3+R_2)$$

$$\eta = \frac{R_2}{R_3+R_2}$$

eq.- 24

2.4 Design method to compensate the stability of feed back loop ; Phase-lag compensation

It is needed to compensate the stability for the feed back loop with a first-order lag element. A first-order lag element has several features what has the amplitude decay with −6dB/Oct and the phase-lag with stopping at 90 deg such as Fig- 3.

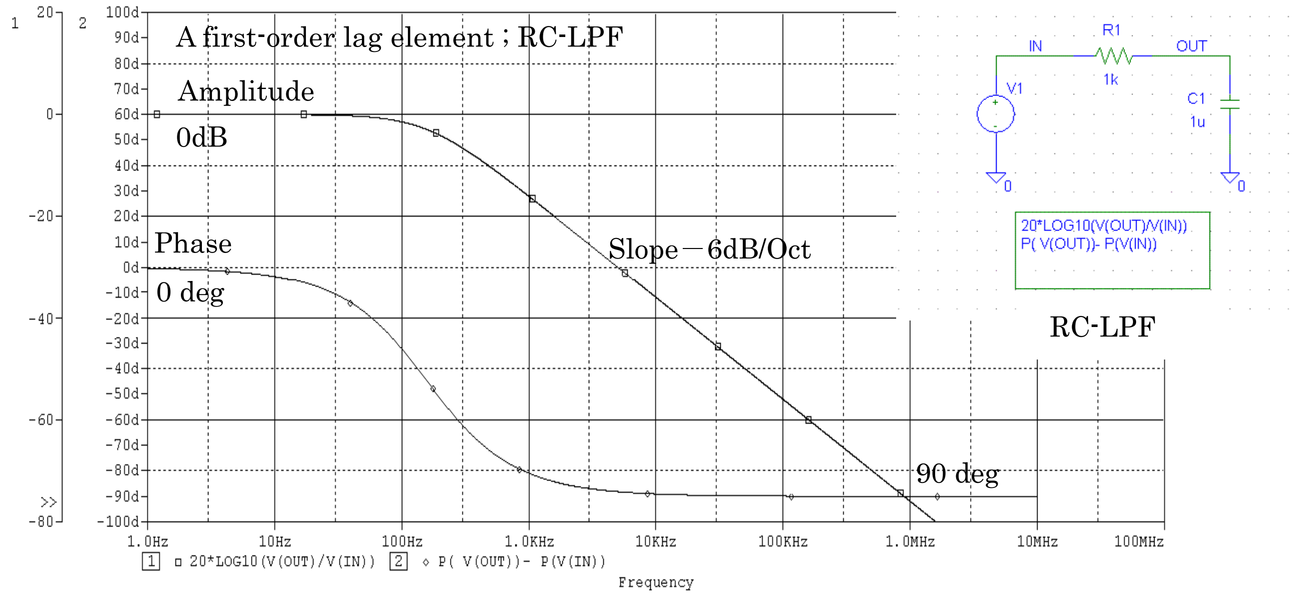

Fig- 3 Transfer characteristics of a first-order lag element RC-LPF

The condition for an instability of feed back loop is what the amplitude value at phase-lag 180 deg region is more than 0dB (>0dB). To the contrary, the condition for an stability of feed back loop is what the amplitude value at phase-lag 180 deg region is less than 0dB (<0dB).

The phase value of total feed back loop at region of DC and sufficient low frequency is 180 deg, that is, what a feed back requires indicates a restoring force. So, the phase-lag 180 deg means the total phase-lag being 360deg in total loop.

What the amplitude value at phase-lag 180 deg region is more than 0dB is indicating that the feed back signal at specific frequency is growing gradually for every go-around on loop cause of being with amplitude >0dB. It is needed for the feed back loop's stability to avoid the conditions what the amplitude value at phase-lag 180 deg region is more than 0dB (>0dB).

The effect of a first-order lag element for the stability of loop is that keeps the amplitude value being less than 0dB via the decay −6dB/Oct in the phase lag 90 deg -stopping region.

It is shown the example characteristics of total feed back loop at Fig- 4 that keeps the condition of stability for total loop having the amplitude < 0dB with sufficient phase margin. We should keep the phase margin being more than 45 deg in order to acquire the stability of loop.

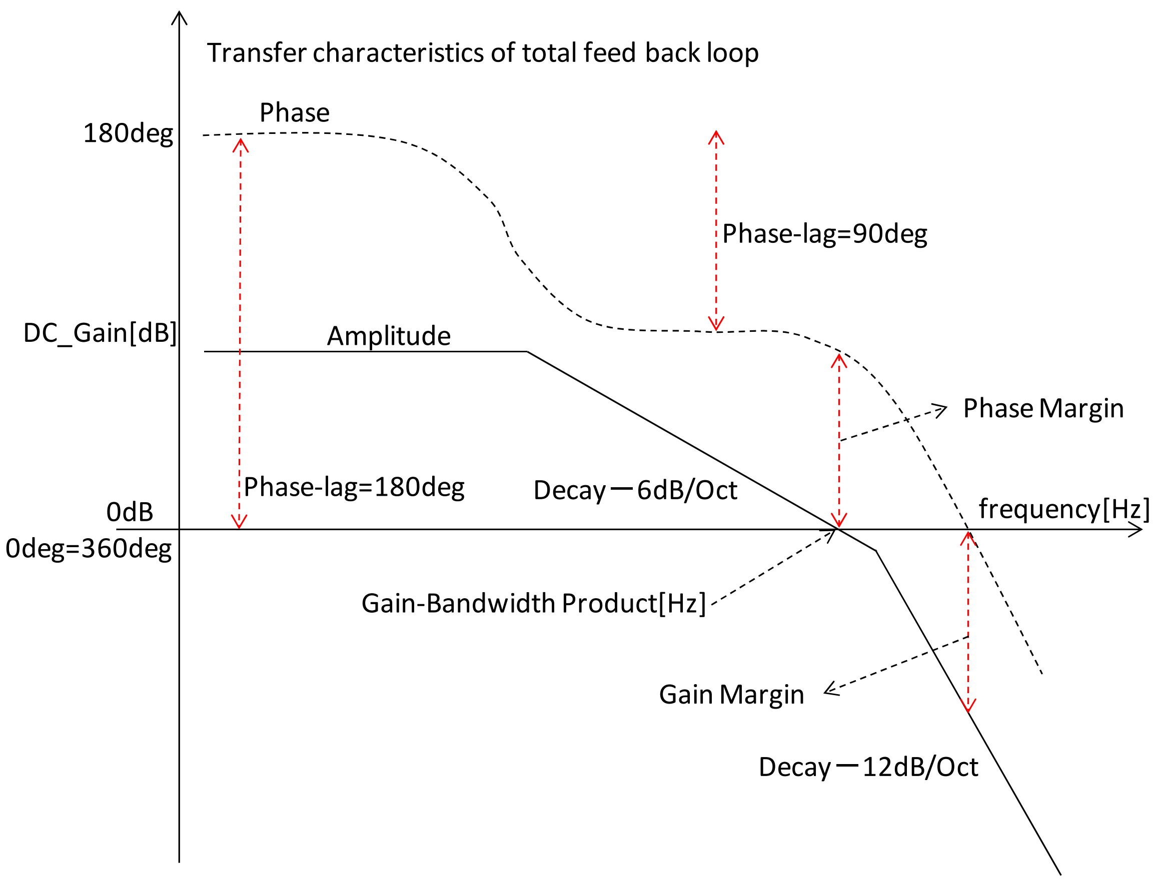

Fig- 4 Transfer characteristics of total feed back loop

2.5 Compressing Error and the gain of total loop

We have close relation between compressing error ε and the gain Gtotal of total loop.

We make the equation about error ε with the gain Gtotal at Fig- 1.

$$\varepsilon(t) = \left(V_{target} - V_{monitor}\right)$$

eq.- 25

$$V_{monitor} = G_{total} \times \varepsilon(t)$$

eq.- 26

Substitute eq.- 25 to eq.- 26.

$$\varepsilon(t) = \left(V_{target} - V_{monitor}\right) = \left(V_{target} - G_{total} \times \varepsilon(t)\right)$$

eq.- 27

Thus,

$$\varepsilon(t) = \frac{V_{target}}{1 + G_{total}}$$

eq.- 28

eq.- 28 means what requires the sufficient large total gain of loop to compress the error value.

It is shown the dead zone of error value and a restoring force at Fig- 5.

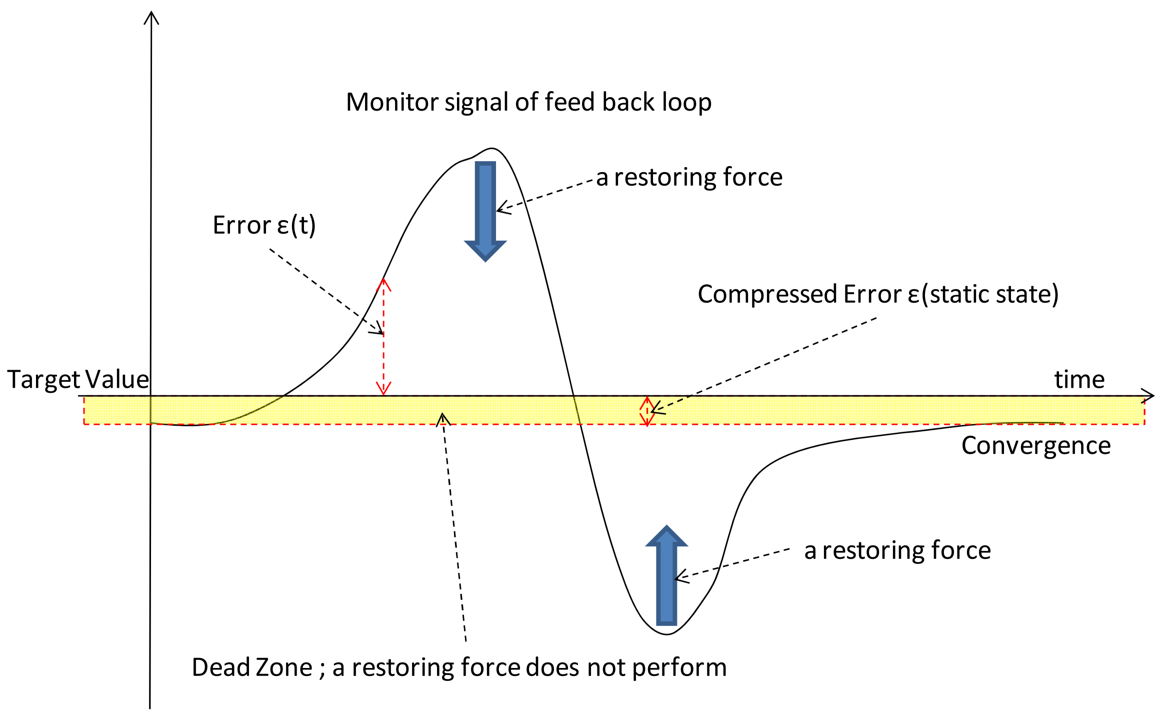

Fig- 5 the dead zone of error value and a restoring force

2.6 Actual design method via Phase-lag compensation and via Phase-lead-lag compensation

The contents of these articles about actual design method via Phase-lag compensation and via Phase-lead-lag compensation by PID filter are paid-services. Please purchase our ***"DSP F28335 Basic Control Platform C-Programming Code Sets"*** .

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