The formula of FBG (Fiber Bragg Grating) characteristics and principle of the differential FBG optical circuit

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1. FBG characteristics
The equations of the FBG wavelength deviations by the strain forces and by the temperature deviations, is shown below.

\[ \frac{\delta \lambda_B}{\lambda_B} = \left( \frac{1}{n} \cdot \frac{\partial n}{\partial T} + \alpha_s \right) \cdot \delta T + G_F \cdot \varepsilon_s \quad \text{(eq.1)} \]

\( \lambda_B \) : filter wavelength of FBG, \( \lambda_B = 1550 \text{[nm]} \), \( n \) : refractive index, \( n = 1.45 \), 
\( \alpha_s \) : temperature coefficient, \( \alpha_s = 0.55 \text{[u-strain/ deg C]} \), 
\( \varepsilon_s \) : external stress strain [u-strain], 
\( GF \) : conversion coefficient of strain, \( GF = 0.78 \), \( \delta T \) : temperature deviation [deg C]

Formula of FBG transfer function : the normalized power transmittance

\[ T = \frac{\cos^2 \left[ \kappa^{(m)} L \sqrt{1 + \left( \frac{\delta^{(m)}}{\kappa^{(m)}} \right)^2} \right]}{1 + \left( \frac{\delta^{(m)}}{\kappa^{(m)}} \right)^2} \quad \text{(eq.2)} \]

Formula of FBG transfer function : the normalized reflection

\[ R = \frac{-\sin^2 \left[ \kappa L \sqrt{1 - \left( \frac{\delta}{\kappa} \right)^2} \right]}{\cos^2 \left[ \kappa L \sqrt{1 - \left( \frac{\delta}{\kappa} \right)^2} - \left( \frac{\delta}{\kappa} \right)^2 \right]} \quad \text{,} \quad \left( \frac{\delta}{\kappa} \right)^2 > 1 \quad \text{(eq.3)} \]

Here,
\( \kappa \) : the coupling coefficient for the 1·st order for unblazed Bragg grating
\[ \kappa = \frac{\pi \eta}{\lambda_B} \]
\( \delta \) : the detuning parameter
\[ \delta = \Omega - \frac{\pi}{\Lambda} \]
\( L \) : the grating length
\[ \Omega = \frac{2 \pi n_{eff}}{\lambda} \]
\( \Lambda \) : period of index modulation
\[ \lambda_B = 2 n_{eff} \Lambda \]
\( m \) : mode index
\( \eta \) : the effective index of guided mode(Clad)
\[ \eta = \Delta n \cdot F \]
the fiber core of Bragg grating
\( \Delta n \): the amplitude of induced refractive index perturbation \( 0.1 \times 10^{-4} \) to \( 0.5 \times 10^{-4} \)
\( \Delta n = n_1 - n_2 \)

\( F \): the fractional modal power in the core

\( V \): Normalized Frequency

\( a \): Core Radius

\( \lambda \): Optical Wavelength

\( \Delta \): relative refractive index difference

\( n_1 \): Core Refractive Index

\( n_2 \): Clad Refractive Index

\( d \): Core Diameter

\( V_c \): Cut-off V-value

\( \lambda_c \): Cut-off Wavelength

- Parameter Example

<table>
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<tr>
<th>( a ) (um)</th>
<th>( d ) (um)</th>
<th>( n_1 )</th>
<th>( n_2 )</th>
<th>( n_1-n_2 )</th>
<th>( \Delta )</th>
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<td>10</td>
<td>1.45</td>
<td>1.448</td>
<td>0.0052</td>
<td>0.36</td>
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\( V = \frac{2\pi}{\lambda} an_1 \sqrt{2\Delta} \)

\( \Delta = \frac{n_1^2 - n_2^2}{2n_1^2} \approx \frac{n_1 - n_2}{n_1} \)

\( V_c = \frac{2\pi}{\lambda_c} an_1 \sqrt{2\Delta} \)

- Simulation Result

Fig-1 Simulation Result of FBG transfer function
2. Characteristics of the differential FBG optical circuit

The equations of the differential FBG optical circuit by the strain forces and by the temperature deviations, is shown below.

\[
\frac{\delta \lambda_{g1} - \delta \lambda_{g2}}{\lambda_{g0}} = \left( \frac{1}{n_1} \frac{\partial n_1}{\partial T} \lambda_{g1} - \frac{1}{n_2} \frac{\partial n_2}{\partial T} \lambda_{g2} \right) \delta T + \alpha_s \left( \frac{\lambda_{g1} - \lambda_{g2}}{\lambda_{g0}} \right) \delta T + G_F \cdot \varepsilon_{s0} \quad \text{(eq.4)}
\]

\[
\lambda_{g0} = \frac{\delta \lambda_{g1} + \delta \lambda_{g2}}{2}
\]

The Optical Circuits of Differential FBG Sensor

- Simulation Result of the differential FBG optical circuit via (eq.2)&(eq.3)
- FBG1&2 Condition : R=80%, BW=0.3nm

![Simulation of FBG Characteristics](image)

**Fig-2** the differential FBG optical circuit

**Fig-3** Transfer characteristics of the differential FBG optical circuit with pre-tension to FBG1 via simulation
The Strain sensing Signal is reflected by FBG1 and going through FBG2.

Fig-5 Simulation of shifted motion for FBG1 wavelength by external compressive stress
The strain sensing signal generates the wavelength deviation of FBG1, and is converted into the optical power level via FBG2. As a result, the strain sensing signal is converted into the optical power level without optical wavelength meter. FBG1 and FBG2 are mutual complementary pair, so the differential FBG optical circuit cancels the wavelength deviation of FBG as to a temperature factor. It is necessary for the operation of the differential FBG optical circuit to use the broadband optical source such as LED, SLED, ASE.

Fig. 6 Strain sensing Signal at optical receiver in the case of Fig. 5

Fig. 7 Simulation result of strain sensing signal via the differential FBG optical circuit
3. Strain Reduction formula of each deviation factor for the differential FBG optical circuit

Basic equation of the differential FBG optical circuit is (eq.4).

1) Strain reduction term as to thermal expansion of FBG

\[ \varepsilon_{\text{FBG-a}} = \frac{1}{G_F} \times \left( \frac{\delta\lambda_{B1} - \delta\lambda_{B2}}{\lambda_{B0}} \right) = \frac{\left( \alpha_{s1} \lambda_{B1} - \alpha_{s2} \lambda_{B2} \right)}{\lambda_{B0}} \cdot \delta T \times \frac{1}{G_F} \]  (eq.5)

2) Strain reduction term as to refractive index deviation between FBG1 and FBG2

\[ \varepsilon_{\text{FBG-n}} = \frac{1}{G_F} \times \left( \frac{\delta\lambda_{B1} - \delta\lambda_{B2}}{\lambda_{B0}} \right) = \left( \frac{1}{n_1} \cdot \frac{\partial n_1}{\partial T} \cdot \lambda_{B1} - \frac{1}{n_2} \cdot \frac{\partial n_2}{\partial T} \cdot \lambda_{B2} \right) \cdot \delta T \times \frac{1}{G_F \cdot \lambda_{B0}} \]  (eq.6)

References