<u>Transfer function of Trans Impedance Amplifier(TIA)</u>

Opto-Electronic Engineering Laboratory Corporation

1. Deviation of TIA transfer function

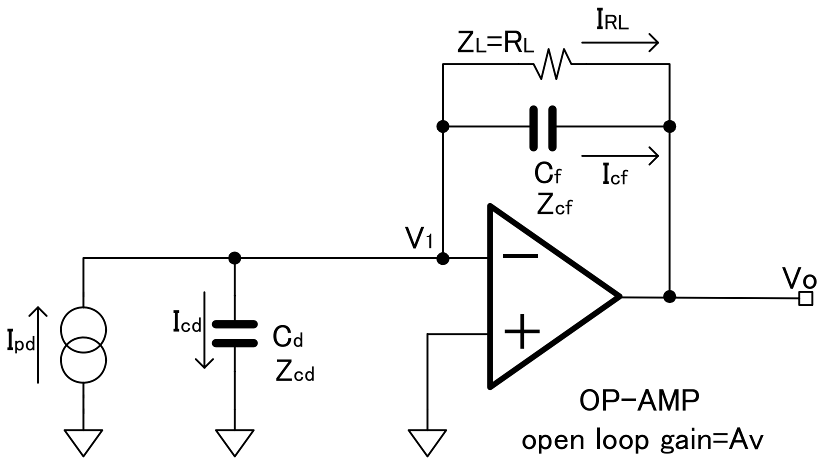

Fig- 1 Schematics of TIA's circuit

The TIA (Trans Impedance Amp) is a circuit that converts a optical current of photodiode into voltage signal with their good features of wide bandwidth and of a low noise. We assume that Ipd is a optical current of photodiode and Cd is a junction capacitance such as Fig- 1.

We write the equations in terms of each current.

$$I_{pd} = I_{cd} + I_{cf} + I_{RL}$$

$$= \frac{V_1}{Z_{cd}} + (V_1 - V_o) \cdot \left(\frac{1}{Z_{cf}} + \frac{1}{Z_L} \right)$$

$$= V_1 \cdot j\omega C_d + (V_1 - V_o) \cdot \left(j\omega C_f + \frac{1}{R_L} \right)$$

$$= V_1 \cdot sC_d + (V_1 - V_o) \cdot \left(sC_f + \frac{1}{R_L} \right)$$

Eq.- 1

Here, ω=2πf, s=jω.

The voltage of output signal at OP-Amp is

$$V_o = -A_V \times V_1$$

Eq.- 2

Substitute Eq.- 2 to Eq.- 1.

$$I_{pd} = V_1 \cdot sC_d + (V_1 - V_o) \cdot \left(sC_f + \frac{1}{R_L} \right)$$

$$I_{pd} = -\frac{V_o}{A_V} \cdot sC_d - \left(\frac{V_o}{A_V} + V_o \right) \cdot \left(sC_f + \frac{1}{R_L} \right)$$

Thus

$$\frac{V_o}{I_{pd}} = \frac{-1}{\frac{1}{A_V} \cdot sC_d + \left(\frac{1}{A_V} + 1 \right) \cdot \left(sC_f + \frac{1}{R_L} \right)}$$

$$= \frac{-A_V}{sC_d + (1 + A_V) \cdot \left(sC_f + \frac{1}{R_L} \right)}$$

$$= \frac{-A_v R_L}{sR_L C_d + (1 + A_v) \cdot (sR_L C_f + 1)}$$

$$= \frac{-A_V}{1 + A_V} \cdot \frac{R_L}{\frac{sR_L C_d}{1 + A_v} + (sR_L C_f + 1)}$$

$$= \left(\frac{-A_V}{1 + A_{Vv}} \right) \cdot \frac{R_L}{1 + sR_L \left(\frac{C_d}{1 + A_v} + C_f \right)}$$

Thus, we get the transfer function of TIA's equations.

$$\hat{G}_{TIA}(V/A) = \frac{V_o}{I_{pd}} = \left(\frac{-A_V}{1 + A_V} \right) \cdot \frac{R_L}{1 + sR_L \left(\frac{C_d}{1 + A_v} + C_f \right)}$$

Eq.- 3

DC gain is

$$DC\_Gain(\Omega) = \left(\frac{-A_{VDC}}{1 + A_{VDC}} \right) \cdot R_L$$

Eq.- 4

2. Calculating the transfer function of TIA with OP-AMP Open Loop transfer function

The transfer function of OP-AMP open loop is

$$A_V = \frac{A_{VDC}}{1 + sT}$$

Eq.- 5

Eq.- 5 is an first-order lag element. OP-Amp keeps the stability of feed-back loop via their characteristics of the first-order lag element.

The GB(Gain-Bandwidth) Product is

$$GB = \frac{A_{VDC}}{2\pi T}$$

Eq.- 6

Substitute Eq.- 5 to Eq.- 3.

$$\hat{G}_{TIA}(V/A) = \left(\frac{-A_V}{1 + A_V} \right) \cdot \frac{R_L}{1 + sR_L \left(\frac{C_d}{1 + A_v} + C_f \right)}$$

$$= \left(\frac{-\frac{A_{VDC}}{1 + sT}}{1 + \frac{A_{VDC}}{1 + sT}} \right) \cdot \frac{R_L}{1 + sR_L \left(\frac{C_d}{1 + \frac{A_{VDC}}{1 + sT}} + C_f \right)}$$

$$= \left(\frac{-A_{VDC}}{1 + sT + A_{VDC}} \right) \cdot \frac{R_L}{1 + sR_L \left(\frac{C_d(1 + sT)}{1 + sT + A_{VDC}} + C_f \right)}$$

$$= \left(\frac{-A_{VDC}}{1+sT+A_{VDC}} \right) \cdot \frac{R_L(1+sT+A_{VDC})}{(1+sT+A_{VDC}) + sR_L\left(C_d(1+sT) + C_f(1+sT+A_{VDC})\right)}$$

$$= \frac{-A_{VDC} R_L}{(1+sT+A_{VDC}) + sR_L C_d(1+sT) + sR_L C_f(1+sT+A_{VDC})}$$

$$= \frac{-A_{VDC} R_L}{s^2(R_L C_d T + R_L C_f T) + s(T + R_L C_d + R_L C_f + R_L C_f A_{VDC}) + (1+A_{VDC})}$$

Thus

$$\hat{G}_{TIA}(V/A) = \frac{-A_{VDC} R_L}{1+A_{VDC}} \cdot \frac{1}{s^2 \cdot \left\{ \frac{TR_L(C_d + C_f)}{1+A_{VDC}} \right\} + s \cdot \left\{ R_L C_f + \frac{T + R_L C_d}{1+A_{VDC}} \right\} + 1}$$

Eq.- 7

Eq.- 7 is an second-order lag element.

3. In the case of flat transfer characteristics and of infinite transfer characteristics

a) In the case of flat transfer and of finite open loop gain characteristics as to OP-Amp
We acquire the limitation of a flat transfer characteristics by T→0 for Eq.- 7.

$$\lim_{T\to 0}\left[\hat{G}_{TIA}(V/A)\right] = \frac{-A_{VDC} R_L}{1+A_{VDC}} \cdot \frac{1}{s \cdot R_L \left\{ C_f + \frac{C_d}{1+A_{VDC}} \right\} + 1}$$

Eq.- 8

Eq.- 8 is an first-order lag element. The cut-off frequency is

$$f_c = \frac{1}{2\pi R_L \left\{ C_f + \frac{C_d}{1+A_{VDC}} \right\}}$$

Eq.- 9

DC gain is

$$DC\_Gain(\Omega) = \frac{-A_{VDC} R_L}{1+A_{VDC}}$$

Eq.- 10

Our "***DSP F28335 Basic Control Platform C-Programming Code Sets***" provides the application samples as to an first-order lag element with Eq.- 8 partially deformed into PID filter. Please see our websales site ;

http://www.optoelec-engineering.com/websale/websale_en.html

b) In the case of flat transfer and of infinite open loop gain characteristics as to OP-Amp

We acquire the limitation of a flat and of infinite open loop gain transfer characteristics by T→0, AVDC→∞ for Eq.- 7.

$$\lim_{T\to 0, A_{VDC}\to\infty}\left[\hat{G}_{TIA}(V/A)\right] = \frac{-R_L}{s\cdot R_L C_f + 1}$$

Eq.- 11

Eq.- 11 is an first-order lag element. The cut-off frequency is

$$f_c = \frac{1}{2\pi R_L C_f}$$

Eq.- 12

DC gain is

$$DC\_Gain(\Omega) = -R_L$$

Eq.- 13

4. About the resonance frequency in the case of a second-order lag element

In the case of a second-order lag element of OP-Amp's transfer function, we obtain the resonance frequency ω_r by Eq.- 7.

$$\omega_r = \sqrt{\frac{1 + A_{VDC}}{TR_L\left(C_d + C_f\right)}}$$

Eq.- 14

We understand what OP-Amp with a large GB Product let the resonance frequency ω_r be high-frequency. Large GB Product is required for fast-TIA's design condition such as Eq.- 15.

$$GB = \frac{A_{VDC}}{T} > \omega_c^2 \cdot \left(C_f + C_d\right)\cdot R_L$$

Eq.- 15

5. Example solution

OP-AMP ; AD8065, GB Product=71MHz, A_{VDC}=446683, T=0.001sec,

R_L=800kΩ、C_f=3pF、C_d=6pF

$$f_c < \frac{1}{2\pi}\sqrt{\frac{A_{VDC}}{T}\cdot\frac{1}{\left(C_f + C_d\right)\cdot R_L}} = 1.2MHz$$